Michael & Jeremy — don't worry, it always looks better on the laser-writer! Rob.
The paper's longer as well — but that's because I've put lots of spacing and widened the margins for the type-setter.

GENERALIZATION OF THE GREENBERGER-HORNE-ZEILINGER ALGEBRAIC PROOF OF NONLOCALITY†

ROBERT K. CLIFTON[1], MICHAEL L. G. REDHEAD[1], and JEREMY N. BUTTERFIELD[2]

We further develop a recent new proof (by Greenberger, Horne, and Zeilinger - GHZ) that local deterministic hidden-variable theories are inconsistent with certain strict correlations predicted by quantum mechanics. First, we generalize GHZ's proof so that it applies to: factorizable stochastic theories; theories in which apparatus hidden-variables are causally relevant to measurement results; and theories in which the hidden-variables evolve indeterministically prior to the particle-apparatus interactions. Then we adopt a more general measure-theoretic approach which requires that GHZ's argument be modified in order to produce a valid proof. Finally, we motivate our more general proof's assumptions in a somewhat different way from previous authors in order to strengthen the implications of our proof as much as possible. After developing GHZ's proof along these lines, we then consider the analogue, for our proof, of Bohr's reply to the EPR argument, and conclude (pace GHZ) that in at least one respect (viz. that of most concern to Bohr) the proof is no more powerful than Bell's. Nevertheless, we point out some new advantages of our proof over Bell's, and over other algebraic proofs of nonlocality. And we conclude by showing that a modified version of our proof still goes through which, like Bell's, does not rely on experimentally unrealizable strict correlations, but still leads to a testable 'quasi-algebraic' locality inequality.

New

"...to admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics". J. S. Bell [1]

1. INTRODUCTION

Within the tradition begun by Bell[1], Greenberger, Horne, and Zeilinger[2] (henceforth GHZ[3]) have recently sketched a simple proof that local deterministic hidden-variable theories are inconsistent with the strict correlations predicted by quantum mechanics (QM) for the following new *Gedankenexperiment* A spin-1 particle, initially in the state m=0 in a magnetic field along the z-axis, decays into two spin-1 particles moving in opposite directions along this axis; and then each subsequently decays into two spin-1/2 particles, one pair (A and B) moving in the +z direction and the

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† It is a pleasure to dedicate this paper to the memory of Professor John S. Bell whose brilliant work over the last three decades did so much to clarify the foundations of quantum mechanics.

1 Department of History and Philosophy of Science, Cambridge University, Free School Lane, Cambridge CB2 3RH UK.

2 Faculty of Philosophy, Cambridge University, Sidgwick Site, Sidgwick Ave., Cambridge CB3 9DA UK.

3 All abbreviations we shall use in this paper are collected in an appendix at the end, with ~~their~~ definitions, for easy reference.

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other (C and D) in the -z direction. After all decays have occurred, and the four particle system is left in the state:

$$|\Psi> = 1/\sqrt{2}\,(|\sigma_z(A) = +1> \otimes |\sigma_z(B) = +1> \otimes |\sigma_z(C) = -1> \otimes |\sigma_z(D) = -1> - |\sigma_z(A) = -1> \otimes |\sigma_z(B) = -1> \otimes |\sigma_z(C) = +1> \otimes |\sigma_z(D) = +1>)$$

the spin of each particle is measured at spacelike separation from the others using spin measurement apparatuses, or 'spin-meters'[4] (for short), with orientations **a**, **b**, **c**, and **d** in the x-y plane perpendicular to the particles' line of flight. Spacelike separation can be arranged by requiring that the particles in each of the pairs (A, B) and (C, D) have different masses, thus making them leave the second spin-1 decay with different velocities. Then, after the particles in a given pair separate sufficiently along the z-axis, spin-meters can be placed in their respective paths to intercept them[5].

The expectation value of the product of the four measured spins along directions **a**, **b**, **c**, and **d**, i.e. $<\Psi|(\sigma\ (A)\cdot\ \mathbf{a})(\sigma\ (B)\cdot\ \mathbf{b})(\sigma\ (C)\cdot\ \mathbf{c})(\sigma\ (D)\cdot\ \mathbf{d})|\Psi>$, can easily be computed using the relations $\sigma_x|\sigma_z=+/-1> = |\sigma_z=-/+1>$; $\sigma_y|\sigma_z=+/-1> = +/-i|\sigma_z=-/+1>$; and Euler's equation to simplify. This yields $-\cos(\theta_A + \theta_B - \theta_C - \theta_D)$, where θ_A [respectively, θ_B, θ_C, and θ_D] is the angle between **a** [**b**, **c**, and **d**] and some arbitrary fixed direction in the x-y plane. To preserve continuity with the Bell's theorem literature, we shall refer to this expectation value as a 'correlation coefficient' between the four spin observables. However, unlike the Bell two-variable *Gedankenexperiment*, it has no direct connection with standard definitions of correlation between greater than two variables[6].

For this *Gedankenexperiment* GHZ start with a local deterministic hidden-variable framework

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[4] We use this generic term rather than 'Stern-Gerlach devices' because the latter are *in principle* incapable of measuring the spin of free elementary particles[3].

[5] To achieve the requisite spatial separation without restriction to the z-axis, one might propose that either: (a) a field be set up to deflect the (A,B) [(C,D)] particles *slightly* away from each other after the second spin-1 decay; or (b) only those particle pairs which, of their own accord, leave the second decay in *slightly* different directions be considered. The advantage of (a) and (b) is that each eliminates the need to move the spin-meters into place before each measurement trial, so that it would be easier to actually perform the experiment. However, since $|\Psi>$ presupposes that all particles move along the z-axis, one would then need to check that with a sufficiently small deviation from this axis, spin-orbit coupling effects will not significantly alter the predictions prescribed by $|\Psi>$. In particular, if the strict correlations that section 3's proof uses are lost, one would have to turn to 5's nonstrictly correlated version of the proof. Alternatively, one could consider measurements on other quantum states (with 3 or more correlated spin-1/2 particles) that do not presuppose a z-axis restriction, but nevertheless predict strict correlations. Since writing this paper, Mermin[4] has elegantly shown that many such states exist for which an informal version of 3's strictly correlated nonlocality proof goes through; and Greenberger *et al.* [5] have studied analogues of these states not involving spin which are experimentally realizable via 'three-particle interferometry'.

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and sketch how a contradiction can be derived between the two-valued 'result functions' required by this framework, and the extremal values, ± 1, of the correlation coefficient, which will obtain when $\theta_A + \theta_B - \theta_C - \theta_D = 0$ or $\pm\pi$. The contradiction is not just statistical (as in Bell's theorem) but 'algebraic', in the sense that values cannot be consistently assigned to all measurement results (determined by the result functions) given the wide range of settings θ_A through θ_D for which $|\Psi\rangle$ predicts strict (i.e., ± 1) correlations. This result, if correct, is surprising in light of the fact that hidden-variabletheories can easily (and consistently!) reproduce all the strict correlations of simpler quantum systems like the spin-1/2 singlet state (cf. Ref. 1, pp. 16 - 17).

New

Interestingly, because they invoke only strict correlations, GHZ appear to claim some philosophical generality over Bell's theorem based on the fact that their proof applies under just those conditions that permit inferences, from certain classical 'locality' and 'reality' assumptions, to the existence of Einstein-Podolsky-Rosen (EPR) (deterministic) 'elements of reality'[7-13]. For given $|\Psi\rangle$'s strict correlations, knowledge of any three measurement results allows one to predict the fourth with certainty; and given a 'no disturbance' locality assumption, one can then apply the well-known EPR sufficient condition so as to assert an element of reality pertaining to the fourth particle. Because of this, GHZ dub strict correlations as 'the super-classical' case; and they claim that because their proof restricts itself just to this case, it is more important than Bell's 'from the point of view of reality'. GHZ's idea seems to be that they show the nonexistence of EPR elements of reality precisely in those cases where their existence is most plausible. But in **4** we shall disagree with this emphasis, arguing that, from the standpoint of Bohr's response to EPR, the existence of elements of reality is neither more nor less plausible in the context of GHZ's proof, than in the context of Bell's, which doesn't confine itself solely to the superclassical case.

In any case, GHZ's claim that their proof goes 'beyond Bell's theorem' is not justified until their result is proved under assumptions at least as weak as those adopted in the most general version of Bell's proof (Ref. 1, Ch. 7). In particular, the latter proof does not rely on determinism - either in the evolution of hidden-variables prior to measurement, or in the sense that they encode intrinsic possessed values for observables; nor on 'faithful measurement' - the assumption that measurement results are solely a product of variables associated with the measured system, i.e. apparatus hidden-variables are causally irrelevant. In contrast, GHZ do not consider apparatus hidden-variables which might be causally relevant to the measurement results; and they explicitly assume that the particle variables at the time of measurement are uniquely determined by those which exist back at the first spin-1 particledecay[6], ruling out evolutionary indeterminism. They also assume that the variables at the time of measurement uniquely determine each measurement result, thus presupposing intrinsic

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6 GHZ attribute this assumption to Bell (Ref. 1, Ch. 2). However, there Bell is neutral as to whether the hidden-variable arguments of his result functions represent the state of the particles when leaving the source, or at some later 'suitable instant' (cf. p. 15).

possessed values or 'elements of reality'.

To be fair, GHZ justify this last assumption by assuming 'locality' (in the sense that results are 'undisturbed' by distant measurements) and using $|\Psi>$'s strict correlations to argue, in EPR fashion, for the existence of elements of reality (just as Bell did in his first nonlocality proof using the singlet state's strict correlations - cf. Ref. 1, Ch. 2, and Bell's comment on p. 157, note 10). However, it is surely desirable to seek a less informal, more cogent, version of EPR's argument to supplement GHZ's - particularly in light of the almost unanimous agreement among recent commentators[12-15] that the EPR argument needs to assume that the particles are 'separable' - a notion not captured (at least unambiguously) by EPR's (and GHZ's) talk of 'no disturbance' between each particle and the distant apparatuses. Our generalization of the GHZ argument to a stochastic hidden-variables framework (i.e. a framework in which elements of reality are not assumed) shall begin, in effect, with a probabilistic formalization of EPR's argument in which an assumption will appear that plays roughly the same role as separability (though, in general, is weaker than it - or so we shall argue). We will also use EPR-type reasoning in a novel way to show that independence of measurement result probabilities from apparatus hidden-variables follows from our proof's locality assumptions and $|\Psi>$'s strict correlations. Finally, we shall define the various hidden-variables that may be causally relevant so as not to presuppose evolutionary indeterminism. Through all of this, we intend to show that GHZ's proof can indeed be made as general as the most general version of Bell's proof (Ref. 1, Ch. 7).

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The overall plan of our paper is as follows. In **2**, we broadly characterize the class of stochastic realist theories that aim to reproduce $|\Psi>$'s correlation statistics; and we motivate the locality principles to be used in a way distinct from previous authors, taking our cue from recent criticisms of these assumptions. In **3**, we give our more general proof of GHZ's result adopting an explicitly measure-theoretic approach. It turns out that, within this approach, the details of GHZ's proof of contradiction (between $|\Psi>$'s strict correlations and the various result functions) need to be modified to produce a valid proof. In **4**, we briefly compare our proof to other algebraic proofs of nonlocality, and then argue that it does not improve on Bell's proof with respect to Bohr's response to EPR. However, we argue that it *does* improve on Bell's proof (particularly, the most general version of that proof, which doesn't invoke any strict correlations - cf. Ref. 1, Ch. 7) in some other respects not discernable from GHZ's own treatment. Finally, in **5**, we show that a modified version of our proof, of roughly the same strength as Bell's, goes through without invoking experimentally unrealizable strict correlations. In fact this modified proof leads to a new 'quasi-algebraic' locality inequality susceptible to experimental test. We conclude that, on balance, our proof compares favourably to other nonlocality proofs at the level of *Gedankenexperiment*; but it remains an open question whether our proof can surpass Bell's in adaptability to experiment.

New phrasing overall

2. THE PROOF'S ASSUMPTIONS IN THE CONTEXT OF A GENERIC REALIST

Broadly speaking, realism is the view that physical systems: (a) exist independently (in particular, exist whether or not they are about to enter measurement interactions); and (b) possess intrinsic properties, or states, which (at least partially) account for measurement results and/or their statistics. ('At least partially' allows for a contribution to measurement statistics from the apparatus' own state.) Such independently existing states we shall henceforth call 'existents' (following Shimony[16]), departing from the traditional term 'hidden-variables'. Our reasons are two-fold: as emphasized by Bell (Ref. 1, p. 92), 'hidden-variable' wrongly suggests that aspects of a system's state can never be revealed through measurements; and that term prohibits, *a priori*, the states attributed by QM itself from being complete objective descriptions which do not need supplementing by as yet unspecified ('hidden') variables.

In our *Gedankenexperiment* consider a particular quadruple of spin measurements undertaken with settings θ_A, θ_B, θ_C, and θ_D which are fixed (not necessarily by 'free-willed' experimenters) *just prior* to the onset of each measurement interaction. Let the existent λ denote the *complete* objective state (or *set* of states) in the *entire* causal history of the four particles prior to these four measurement interactions. (The term 'causal' allows either stochastic or deterministic causation.) This very general characterization of λ allows for the possibility that the particles' state: (a) evolves indeterministically in time prior to the measurements; (b) fails to encode determinate values for all (or any) observables; (c) fails to be spatially (or temporally) separable into a conjunction of determinate states pertaining to each of the four particles (or to each of the times prior to measurement); and (d) fails to satisfy the Markov property that the probability of an event (e.g., the registration of each measurement result) depends only on the state(s) immediately preceding it in time and not also on earlier states (e.g. of the measured, or measuring, system). (For a brief discussion of theories in which (d) holds, cf. Ref. 17, and references therein.)

Similarly without loss of generality, let the existent $\mu_A[\mu_B, \mu_C, \mu_D]$ denote the complete (set of) objective state(s) of the A [B, C, D] spin-meter and its associated particle detection apparatus, in their entire causal history prior to the A [B, C, D] measurement. Each $\mu_A[\mu_B, \mu_C, \mu_D]$ will be assumed to include the macroscopic orientation θ_A [θ_B, θ_C, θ_D] fixed *just before* the interaction with the A [B, C, D] particle, however the latter is an important parameter which we shall refer to explicitly.

The spacetime regions with which these various existents would be associated for a typical '4-measurement' are depicted in figure 1, where we have assumed that the (A, B) [(C, D)] apparatuses intercept their respective particles from opposite sides of the z-axis in the x-z plane in order to minimize the possibility that their causal histories overlap. (More on this point, and other details of figure 1, below.)

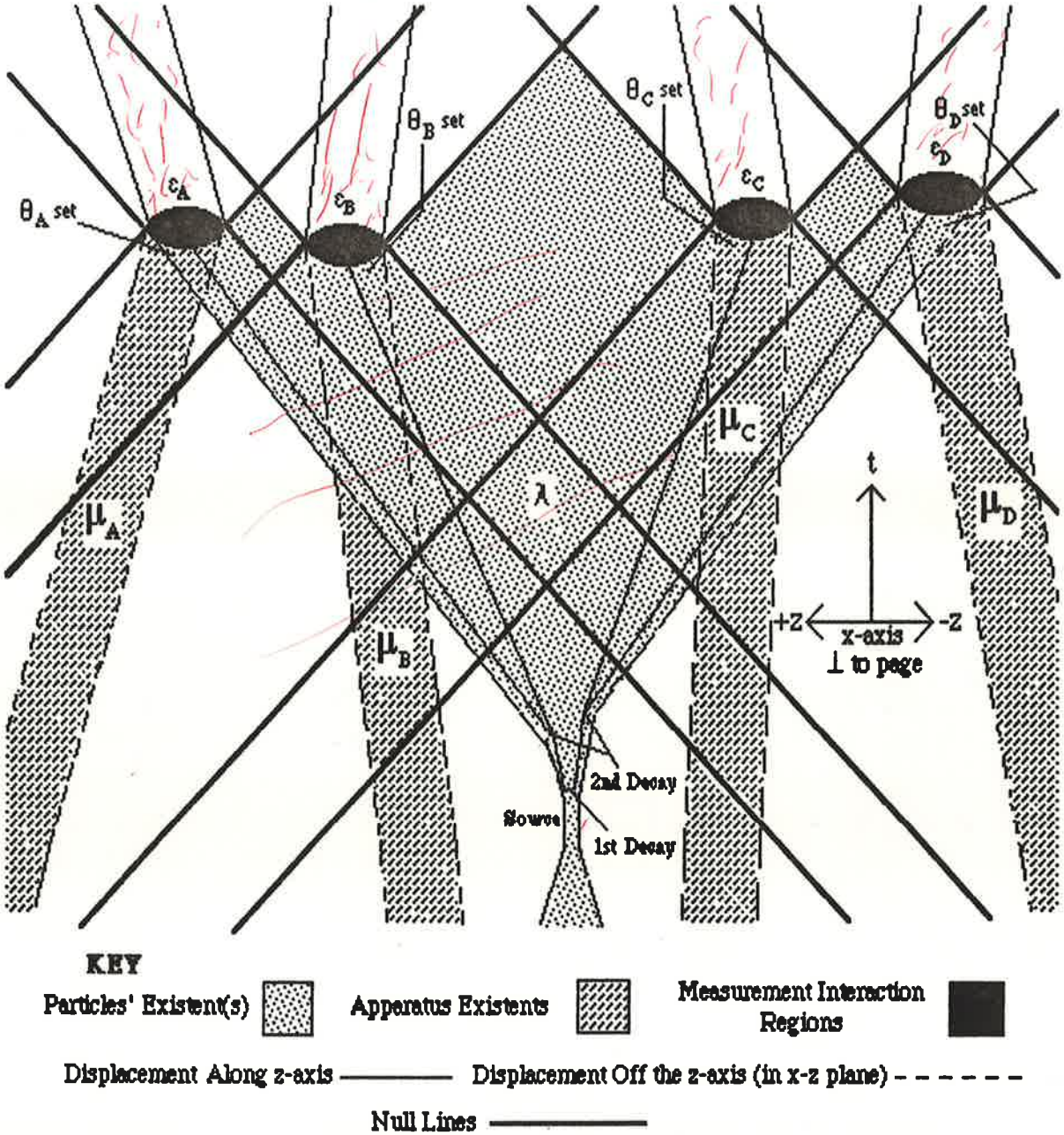

Figure 1 - Spatio-Temporal Regions for Existents in a Generic Realist Theory

Let $P(\varepsilon_A,\varepsilon_B,\varepsilon_C,\varepsilon_D/\lambda,\mu_A,\mu_B,\mu_C,\mu_D,\theta_A,\theta_B,\theta_C,\theta_D)$ be the joint probability that the four measurement results ε_A through ε_D (each = ±1, in units of $h/2\pi$) are obtained[7], given the existents λ and μ_A through μ_D, and the settings θ_A through θ_D. To minimize notational complexity we define $\varepsilon \equiv \langle\varepsilon_A,\varepsilon_B,\varepsilon_C,\varepsilon_D\rangle$, $\mu \equiv \langle\mu_A,\mu_B,\mu_C,\mu_D\rangle$, and $\theta \equiv \langle\theta_A,\theta_B,\theta_C,\theta_D\rangle$, so that the this probability reduces to just $P(\varepsilon/\lambda,\mu,\theta)$.

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[7] The 'results' ε_A through ε_D can represent states of the apparatus 'pointers' and/or the particles after the 4-measurement is completed. Despite Schrödinger's cat, our assuming these results are (macroscopically) distinct from eachother surely does not compromise the generality of our proof. For the realist will resist the superposition of macroscopically distinguishable states in favour of e.g. spontaneous wave packet reduction theories[18], or many-worlds theories[19].

In general, for fixed θ the existents will vary from one 4-measurement to another for two reasons. First, because their preparation is assumed to be (at least at present) uncontrollable. Second, because they are defined as states associated with a spacetime region outside the four measurement interaction regions and their future lightcones, and the temporal ordering (in the lab frame) of these latter regions may vary from one 4-measurement to another.

Thus for consistency with the 4-measurement correlation coefficient dictated by $|\Psi>$, the realist must recover it, in particular its extremal values, by averaging the corresponding correlation coefficient prescribed by the existents, using some density distributions representing ignorance about which existents obtain on a particular occasion:

Strict Correlation Recovery Integral (SCRI):

$$\int_\Lambda \left[\int_M \left(\sum_\varepsilon \Pi_\varepsilon P(\varepsilon/\lambda,\mu,\theta) \right) \rho(\mu/\lambda,\theta) d\mu \right] \rho(\lambda/\theta) d\lambda = \pm 1 \quad \forall \theta \in \Theta_\pm$$

In SCRI, Π_ε denotes the product $\varepsilon_A\varepsilon_B\varepsilon_C\varepsilon_D$ of the elements of the quadruple ε; $\Theta_+ \equiv \{\theta | \theta_A + \theta_B - \theta_C - \theta_D = \pm\pi\}$; and Θ_- is defined similarly with 0 replacing $\pm\pi$. Also Λ is the set of all possible λ compatible with at least one setting quadruple in Θ (where Θ is the union $\Theta_+ \cup \Theta_-$); and M is the set of all possible μ compatible with at least one possible λ and setting quadruple in Θ. Measurable subsets of Λ and M and measures on them must be defined; and as usual we will suppose, only for simplicity, that our probability measures are absolutely continuous with respect to the basic measure, and so given by densities. But *a priori* we impose no substantive restrictions on the sets Λ and M, or their measures. In particular, we allow the existent distributions in SCRI to depend upon the meter settings θ; and on other controllable macroscopic parameters associated with the source and detectors (which we assume to be fixed $\forall\theta \in \Theta$, and thus omit from our notation).

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Further, without loss of generality we can regard the set M as the Cartesian product $M_A \times M_B \times M_C \times M_D$, where M_A is the set of all μ_A, such that μ_A is compatible with at least one possible triple of apparatus existents $\mu_{B,C,D}$ (not necessarily compatible with all such triples!); and M_B - M_D are defined similarly. Of course this means that, in general, M may include quadruples, μ, that are incompatible with eachother. However, whenever a certain combination of apparatus existents is incompatible, the density, $\rho(\mu/\lambda,\theta)$, of such a combination must also be zero, leaving SCRI's integral unchanged from what it would have been had we not assumed M to be a Cartesian product. So our choice to define M as such a product really is just a simplifying convention with no physical implications for the realist's recovery of QM's strict correlations. Further, when SCRI's integral is evaluated, it doesn't matter how probabilities like $P(\varepsilon/\lambda,\mu,\theta)$, that may happen to be conditional on incompatible combinations of existents, are defined, or what values these take - for, because incompatible combinations have zero density, our choice of how to define them (or what values they have) will simply never show up after evaluating any integral. In section 3's proof we shall exploit this freedom, and the freedom (without loss of generality) to regard $\mu_{A,B,C,D}$ as independent variables

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throughout M (which is entailed by the latter being a Cartesian product).

We turn now to the assumptions which our proof will show the realist must reject given the condition of recovery expressed in SCRI. There are only three, the first two being conditions about probabilities - specifically, about measurement results being statistically independent of distant facts, be they results, settings, or apparatus existents:-

OutcomeFactorability (OF): $P(\varepsilon/\lambda,\mu,\theta) = P(\varepsilon_A/\lambda,\mu,\theta)\,P(\varepsilon_B/\lambda,\mu,\theta)\,P(\varepsilon_C/\lambda,\mu,\theta)\,P(\varepsilon_D/\lambda,\mu,\theta)$

Outcome Locality (OL): $P(\varepsilon_A/\lambda,\mu,\theta) = P(\varepsilon_A/\lambda,\mu_A,\theta_A)$; $P(\varepsilon_B/\lambda,\mu,\theta) = P(\varepsilon_B/\lambda,\mu_B,\theta_B)$

$P(\varepsilon_C/\lambda,\mu,\theta) = P(\varepsilon_C/\lambda,\mu_C,\theta_C)$; $P(\varepsilon_D/\lambda,\mu,\theta) = P(\varepsilon_D/\lambda,\mu_D,\theta_D)$

In OF and OL we follow the axioms of standard probability theory so that $P(\varepsilon_A/\lambda,\mu,\theta) = \sum_{\varepsilon_B,\varepsilon_C,\varepsilon_D=\pm1} P(\varepsilon/\lambda,\mu,\theta)$, etc. Further, the notation used on the right-hand sides in OL denotes the common value of the left-hand side as the existents and settings which do not appear on the right vary throughout M and Θ.

OF and OL parallel two similar 'locality' assumptions commonly adopted in Bell's theorem[20]; so we shall not rehearse their standard motivation in detail here - suffice it to say that they are plausible given that:

(I) Each measurement is undertaken at spacelike separation from the other three;
(II) The causal history of each measurement device is sufficiently disentangled from the other three and from the particles in the backward lightcones of the measurement events; and
(III) Spacelike causation does not exist.

OF is the assumption that we alluded to in **1** which plays the role of EPR's tacit 'separability' assumption in their argument. However, OF is in general weaker than separability (which we do not need to assume), if we adopt the most natural definition of the latter as asserting that the composite state of the four particles, λ, is reducible to separate states for each particle individually which determine their composite state. True, in the particular case where we assume the state vector of QM exhaustively describes the state of the particles, λ, OF and separability coincide. For it is a well-known theorem of QM that conditions like OF hold if, and only if, the quantum state is a product (or, 'factorable') state, i.e. a state which permits the individual states of the component systems to be 'disentangled' (to use Schrödinger's term). So, for example, both OF and separability fail if λ is just $|\Psi>$.

New

However in the case of the de Broglie-Bohm causal interpretation[21], which does not take QM's state ascriptions to be exhaustive, OF holds while separability fails. In its simplest form, their interpretation posits a λ comprising the positions of our four spin-1/2 particles and $|\Psi>$, which

generates a field collectively guiding the four particles. But since $|\Psi>$ is manifestly nonfactorable, λ is *not separable* into parts which pertain to, or act upon, each particle separately. (In fact, this nonseparability is exactly what forces their theory to violate OL once $|\Psi>$ becomes modified by its interaction with the spin-meters - cf. Ref. 1, p. 11.) On the other hand, λ fixes measurement results deterministically, which implies that OF must hold. So the causal interpretation provides an excellent example where OF holds while separability fails, demonstrating that they are not the same[8].

New (Almost!)

In addition to motivating OF and OL, (I)-(III) motivate the following assumption concerning the particles' existent distribution:

Particle Locality (PL):

$$\rho(\lambda/\theta) = \rho(\lambda) \quad \forall\lambda,\theta \in \Theta$$

Here again, the notation used on the right-hand side denotes the common value of the left-hand side as the settings which do not appear on the right vary throughout Θ. PL permits us to define setting-independent $\rho(\lambda)$-measures $\forall$ subsets S of Λ: $m_\lambda[S] \equiv \int_S \rho(\lambda)d\lambda$.

We now define 'Total Factorability' (TF) as the conjunction of OF and OL. And we define:

$$\Lambda_{TF} \equiv \{\lambda | \forall\mu\in M, \forall\theta\in\Theta, \forall\varepsilon\text{: TF holds}\}$$

Existent sets such as Λ_{TF}, over which 'locality' assumptions are true, are not usually explicitly invoked in proofs of nonlocality. But we need Λ_{TF} in order to make plain in **3** why GHZ's own proof no longer goes through in our more general measure-theoretic framework; and to facilitate a clear and complete statement of our theorem - which will be that PL and SCRI require the m_λ-measure of Λ_{TF} to be 0. In effect: with measure 1, TF must fail.

But before proving the theorem it is worth considering two important criticisms of assumptions like ours above; though they have been raised in the context of Bell's theorem, they are also applicable here. Our aim is to rebut these criticisms, and so keep the above assumptions plausible, by carefully defining the particle and apparatus existents which appear in our assumptions (cf. figure 1).

The first criticism, raised by Jarrett[23], questions the stochastic independence of measurement results when their probabilities are conditioned only on 'initial' existents, call them μ_0 and λ_0, encoding the state of the particles and apparatuses upto some time well before measurement (e.g. at the time of the first spin-1 decay). Spelling out this criticism will help us justify why our OF

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[8] Note that our sense of separability is *not* Howard's (Ref. 22, p. 230, note 13) who chooses to define separability of the A - D particle states in a way which would render this notion *equivalent* to OF. The case of the causal interpretation shows how Howard's terminology is misleading.

conditions on μ and λ (which encode states right upto measurement) rather than μ_0 and λ_0. For brevity, call the disputed assumption OF_0, for 'OF with probabilities conditional on (μ_0,λ_0)'. And let $\nu_0 = (\mu_0,\lambda_0)$ and $\nu = (\mu,\lambda)$, so that we can discuss both the apparatus and particle existents at once. Then the criticism is that OF_0 can trivially fail for realist theories in which the existents evolve indeterministically upto measurement (specifically, theories which predict that for at least two ν-existents (compatible with ν_0), say ν' and ν'', $1>P(\nu'/\nu_0,\theta)>P(\nu''/\nu_0,\theta)>0$). For conditioning on a distant result can alter the probability of a nearby result by giving information about how ν_0 evolved[9].

Admittedly, this criticism holds only if the information is probabilistically relevant to both results. This means the theory must either: prohibit the separation of each ν-existent into parts, ν_A through ν_D, causally relevant to each measurement interaction separately; or permit this, but allow ν_A through ν_D to causally interact with each other, or with distant results, in ways already ruled out by our motivating assumptions (I)-(III). So one may wonder why we circumvent this criticism of OF_0 by assuming the weaker OF, rather than additional separability and locality assumptions.

One reason is that OF's conditionalization on more than just the existents at the time of the first decay avoids some recently proposed nonquantum mechanical counterexamples[26,27]: for details, cf. Ref. 28. But our main reason is that, although additional locality/separability assumptions are classically plausible, they unnecessarily weaken our theorem's conclusion. Thus in figure 1 we have not separated the particles' existent, λ, into parts pertaining to the individual particles. So we allow the particles to be 'entangled', as in the special case where $\lambda = |\Psi>$: after all, they (unlike the apparatuses) directly interact with eachother prior to measurement. Further, if a realist theory does ascribe separate states to each particle, figure 1's λ allows for the possibility of causal influences between these states at any time prior to the 4-measurement. Indeed, it even allows for states of the *source* at the time of measurement (i.e. well after the first decay) to influence ε and the particles: for these states will lie within figure 1's central 'diamond' bounded by null lines between the B and C measurement regions. Allowing λ to 'spread into' this region, and into the two other diamonds between the A/B and C/D measurements, does not affect our proof, nor the plausibility of PL. (As long as λ *excludes* each diamond's future boundary (composed of 2 null lines), PL will not fail by part of λ being in the forward lightcone of the fixing of a meter setting.)

The second criticism, originating in a debate between Shimony *et al*[29] and Bell[30], centres around PL. In the most general version of his theorem (Ref. 1, Ch. 7), Bell divides existents into: those in the overlap of the backward lightcones of the measurement events; and those in the remainder

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[9] A similar argument has been widely used to motivate OF (and even OF_0!) on the grounds that its probabilities are conditional on *all* the details that might be causally relevant to the measurement results (cf. Refs. 1, p. 54; 24, p. 206; and 25, p. 579).

of these backward lightcones. The difficulty is that Bell takes the overlap existents to be the total states of the spacetime region so that they contain information not restricted to the particles' initial state and preparation, and can hardly fail to influence the eventual setting choices θ. So his version of PL, which pertains to *all* of these overlap existents, is implausible. To avoid this, Shimony *et al.* opt for two hypotheses: that the set of overlap existents causally relevant to the measurement results ε are just those which are intrinsic to or in the immediate spatio-temporal vicinity of the particles; and that their distribution can be regarded as a marginal distribution averaged over the extra overlap existents that Bell includes. The idea of the first hypothesis seems to be that if one is more specific about which overlap existents are causally relevant to ε, then a backwards causal entanglement between these existents and θ is less credible. The second hypothesis weakens this even further by allowing that factors causally relevant to θ in the overlap *do* influence the particles' existent, but that when averaged out they leave its distribution independent of θ. Although these hypotheses restrict the scope of the theorem, Shimony *et al.* claim that they increase its plausibility since they are methodological hypotheses 'no stronger than one needs for experimental reasoning generically' (cf. Refs. 31 and 32 for further arguments which supplement theirs).

We propose to follow Shimony *et al.* Although we differ from them in our defining existents right upto measurement, we take our λ-existents to be intrinsic to the particles - which is why, in figure 1, λ's region does not completely fill any measurement event's backwards lightcone, nor their intersection. (Of course, exactly which existents are 'intrinsic' to the particles depends upon the realist theory at issue, so in this respect figure 1's λ provides only a guide. But, at a minimum, λ should include the classically expected trajectories of the particles, which figure 1 indicates.) Further, we regard ρ(λ) as averaged over other factors relevant to θ in the overlap of the four backward lightcones

A similar point can be made about the other existents causally relevant to ε being intrinsic to the apparatuses (as with our μ), rather than total states of spacetime regions; and treating ρ(μ/λ,θ) as a distribution averaged over these states. However, since in **3**'s strictly correlated proof we do not need to assume that the apparatus distribution is independent of θ, there is not as much motivation to do this. But in both Bell's and our nonstrictly correlated proofs (discussed in **4** and **5**, respectively) there is such motivation, since there ρ(μ/λ,θ) is assumed to factor into a product of distributions pertaining to each apparatus existent and setting separately.

Having accommodated the Jarrett and Shimony *et al.* criticisms, figure 1's spacetime diagram turns out to be very different from Bell's (cf. Ref. 1, p. 55, fig. 3). Nevertheless we shall retain figure 1's existents even when discussing Bell's proofs in **4**. (For an alternative approach, Ref. 28 surveys ways in which, adopting Bell's method of taking existents to be total states of spacetime regions, his assumptions can be made as plausible as possible.)

New

It is worth discussing one final doubt Shimony (Ref. 33, p. 226(note)) has since advanced

as to whether the existents associated with the 4 apparatuses can be *fully* causally disentangled in any individual measurement trial. For even if an apparatus existent is sufficiently intrinsic (dispelling the concerns of Shimony *et al.*), Shimony argues that it 'is an enormously complicated and detailed compendium of information' that seems bound to leave 'some small trace' upon the others, even without any spacelike causation. If Shimony's doubt is correct, this would have the effect of making the apparatus existents so strongly entangled, that no two quadruples of existents, μ and μ', appearing in different measurement trials could share a common A, B, C, or D apparatus existent without at least one of them having zero measure (because it would have to be an incompatible combination of existents!). And this would render OL physically empty.

For example, consider the equation $P(\varepsilon_A/\lambda,\mu,\theta) = P(\varepsilon_A/\lambda,\mu_A,\theta_A)$ that appears in OL. The right-hand side probability was defined to be the common value of the left-hand probability as the apparatus existents (and settings) not appearing on the right vary. Now if μ_A figures in only one compatible quadruple μ, then the variation of apparatus existents not appearing on the left can only lead to the consideration of probabilities conditional on incompatible existent combinations. (Explicitly, the above equation will imply that $P(\varepsilon_A/\lambda,\mu,\theta) = P(\varepsilon_A/\lambda,\mu',\theta)$ for all incompatible μ' containing μ_A.) And since these probabilities have no physical content (and can be defined in any way one chooses without affecting our argument, as noted earlier), OL will inherit this lack of physical content.

There are two strategies one can use to deal with Shimony's doubt. First, one could grant that each apparatus existent leaves *some* trace on the others, but hold that there is no reason to think that such traces will be anything but small. So, for example, μ_A will in general carry a small amount of information about the other existents $\mu_{B,C,D}$ - call this information $\mu_A^{B,C,D}$. Since it is small, it is plausible to suppose that it will be effectively causally irrelevant to the probability for any A-measurement result, ε_A, i.e. $P(\varepsilon_A/\lambda,\mu_A,\theta_A) = P(\varepsilon_A/\lambda,\mu_A - \mu_A^{B,C,D},\theta_A)\ \forall\mu_A$, where '$\mu_A - \mu_A^{B,C,D}$' is the A apparatus existent, less any small traces of information from the other existents. Given this, we can redefine each apparatus existent so that it leaves out any causally irrelevant information about the other existents (i.e. redefine μ_A as $\mu_A - \mu_A^{B,C,D}$; μ_B as $\mu_B - \mu_B^{A,C,D}$, etc.); after all, this extra small amount of information should make no significant difference to the probabilities! This strategy restores the possibility that a given existent can be compatible with more than one combination of the other three existents, and so restores the physical content of OL.

The other way is to follow Shimony's strategy in Ref. 33, and subsequent publications[34], and drop TF in favour of the weaker assumptions:

Outcome Independence: $\underline{P}(\varepsilon/\lambda,\theta) = \underline{P}(\varepsilon_A/\lambda,\theta)\,\underline{P}(\varepsilon_B/\lambda,\theta)\,\underline{P}(\varepsilon_C/\lambda,\theta)\,\underline{P}(\varepsilon_D/\lambda,\theta)$

Parameter Independence: $\underline{P}(\varepsilon_A/\lambda,\theta)=\underline{P}(\varepsilon_A/\lambda,\theta_A)$; $\underline{P}(\varepsilon_B/\lambda,\theta)=\underline{P}(\varepsilon_B/\lambda,\theta_B)$; $\underline{P}(\varepsilon_C/\lambda,\theta)=\underline{P}(\varepsilon_C/\lambda,\theta_C)$; $\underline{P}(\varepsilon_D/\lambda,\mu,\theta)=\underline{P}(\varepsilon_D/\lambda,\theta_D)$

New!

! Check!

where the probabilities, $\underline{P}$, are averaged over the μ existents as:

e.g. $\underline{P}(\varepsilon/\lambda,\theta)=\int_M P(\varepsilon/\lambda,\mu,\theta)\rho(\mu/\lambda,\theta)d\mu$; $\underline{P}(\varepsilon_A/\lambda,\theta)=\int_M[\sum_{\varepsilon_B,\varepsilon_C,\varepsilon_D=\pm1}P(\varepsilon/\lambda,\mu,\theta)]\rho(\mu/\lambda,\theta)d\mu$, etc.

The conjunction of the above two assumptions is equivalent to a μ-averaged version of TF, call it '$\underline{TF}$'. And it can easily be checked that $\underline{TF}$ is sufficient for our theorem in **3**; just as it is for Bell's - the case Shimony is concerned with. So by using $\underline{TF}$ rather than the stronger TF, which in fact implies $\underline{TF}$, given $|\Psi>$'s strict correlations (cf. **3**, Lemma 2), we can allow some scepticism about TF, in particular about OL's physical content, and yet still derive a contradiction with QM's predictions.

New)

We prefer the first strategy to Shimony's for three reasons. First, by using TF rather than $\underline{TF}$ in the next section, it becomes possible to show (cf. Lemma 2) that TF implies $\underline{TF}$, given $|\Psi>$'s strict correlations - a result which sheds new light on the consequences of denying Shimony's $\underline{TF}$ (which has been surprisingly little discussed in the literature, nor even distinguished from TF!). Second, as so often, there is a trade-off between using premisses as weak as possible for the task at hand, like Shimony's $\underline{TF}$, and using premisses for which a simple physical motivation can be found, like TF. Being a trade-off it's a matter of judgement: ours is that TF is the best assumption to use because it is manifestly locality-motivated as soon as one adopts the first strategy above. Third, if the first strategy is rejected, it is not clear that denying Shimony's $\underline{TF}$, which entails denial of TF (i.e. OL or OF), makes the realist's position undesirable. For as we've said, the realist can then be sceptical about the physical implications of denying OL. Since Shimony offers no other sufficient conditions for $\underline{TF}$ which the realist would be loathe to deny, a proof that shows $\underline{TF}$ to be in contradiction with QM's predictions would have questionable import for the realist.

New

3. GENERALIZED ALGEBRAIC PROOF OF NONLOCALITY

Having introduced some notation and motivated our assumptions, we now prove the main theorem: that given PL, and SCRI (the constraint of compatibility with $|\Psi>$'s strict correlations), $m_\lambda[(\Lambda_{TF})'] = 1$ (where $\forall$ subsets S of Λ, S' denotes the set theoretic complement of S). That is: the measure of particle existents which violate either of our 'locality' assumptions (Outcome Factorability or Outcome Locality), for at least one quadruple μ of apparatus existents (and so settings θ), and one quadruple ε of measurement results consistent with the strict correlations (or anti-correlations) of our *Gedankenexperiment* must be 1. After proving this, we shall explain why GHZ's own proof no longer goes through in our more general measure-theoretic framework.

We begin with a lemma. Let $\mathbf{I}(\lambda,\theta)$ be a shorthand for SCRI's apparatus existent integral (in square brackets in **2**). Let $\Theta^c_\pm$ be countable subsets of $\Theta_\pm$, respectively. We need not commit ourselves to *which* countable subsets $\Theta^c_\pm$ represent until the final stage of the proof below. (In fact,

to do so at this stage would not only make our notation more cumbersome, but fail to bring out the fact, emphasized below, that many choices for $\Theta^c_{\pm}$ are sufficient for the final stage of our proof below - a fact which strengthens our proof.) Let $\Lambda^c_{SC} \equiv \{\lambda | \mathbf{I}(\lambda,\theta) = +1, \forall\theta\in\Theta^c_+ \text{ and } \mathbf{I}(\lambda,\theta) = -1, \forall\theta\in\Theta^c_-\}$. So Λ^c_{SC}'s superscript and subscript indicate that countably ('c') many of the strict correlations and anti-correlations ('SC') predicted by $|\Psi>$ are mirrored by each $\lambda \in \Lambda^c_{SC}$.

Lemma1: - $\quad PL \wedge SCRI \Rightarrow m_\lambda[(\Lambda_{TF})'] = 1 - m_\lambda[\Lambda_{TF}\cap\Lambda^c_{SC}]$

Proof 1 - It is easy to show that SCRI implies $\forall\theta\in\Theta_{\pm}$, $m_\lambda[\{\lambda|\mathbf{I}(\lambda,\theta) = \pm1\}] = 1$ (for either upper or lower signs). Using the following theorem of probability theory:

$$m_\lambda[S_1] = m_\lambda[S_2] = \ldots = m_\lambda[S_n] = \ldots = 1 \Rightarrow m_\lambda[S_1\cap S_2\cap \ldots S_n\cap \ldots] = 1$$
$$\forall \textit{ countable} \text{ sequences of sets } S_1, S_2, \ldots S_n, \ldots \qquad (3.1)$$

it follows that $m_\lambda[\Lambda^c_{SC}] = 1$. Again using probability theory:

$$m_\lambda[(\Lambda_{TF})'] = 1 - m_\lambda[\Lambda_{TF}\cap\Lambda^c_{SC}] - m_\lambda[\Lambda_{TF}\cap(\Lambda^c_{SC})'] \qquad (3.2)$$

But since $m_\lambda[(\Lambda^c_{SC})'] = 0$, the last term on the right in (3.2) is 0. QED.

For λ within $\Lambda_{TF}\cap\Lambda^c_{SC}$, let us define the (in general, stochastic) 'result' function $\mathbf{A}(\lambda,\mu_A,\theta_A) \equiv P(+1/\lambda,\mu_A,\theta_A) - P(-1/\lambda,\mu_A,\theta_A)$ which encodes the expectation value of A's spin for given existents (similarly for **B**-**D**). The right-hand side is well-defined: recall that within $\Lambda_{TF}\cap\Lambda^c_{SC}$, A-probabilities depend on μ only via μ_A and so on θ only via θ_A, etc. The next lemma shows that there must also be no μ_A-dependence: so we will be able to write $\mathbf{A}(\lambda,\theta_A)$, etc.

Lemma2: - Let $\lambda\in\Lambda_{TF}\cap\Lambda^c_{SC}$. Then:

(1) $\forall\theta\in\Theta^c_+\cup\Theta^c_-$, $\mathbf{A}(\lambda,\mu_A,\theta_A)$ takes a common value $\mathbf{A}(\lambda,\theta_A)$ $\forall\mu_A\in M$; and similarly for **B**-**D**.

(2) $\forall\theta\in\Theta^c_+$, these common values obey $\mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D)=+1$; and

(3) $\forall\theta\in\Theta^c_-$, they obey $\mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D)=-1$.

(Note that there is no immediate contradiction between the equations in (2) and (3), since $\Theta^c_+\cap\Theta^c_-=\varnothing$. However, in the main theorem to follow we show that there exists a choice of Θ^c_+ and Θ^c_-, in fact many such choices, which imply a contradiction between the values assigned to one of the four terms in common between the two equations appearing in (2) and (3).)

Proof 2 - First note that each $\lambda\in\Lambda_{TF}\cap\Lambda^c_{SC}$, by the latter set's very definition, must satisfy:

$$\forall\theta\in\Theta^c_{\pm},\ \mathbf{I}(\lambda,\theta) = \int_M \mathbf{A}(\lambda,\mu_A,\theta_A)\mathbf{B}(\lambda,\mu_B,\theta_B)\mathbf{C}(\lambda,\mu_C,\theta_C)\mathbf{D}(\lambda,\mu_D,\theta_D)\rho(\mu/\lambda,\theta)d\mu = \pm1 \quad (3.3)$$

where for each λ, (3.3) must hold for both upper and lower signs (by Λ^c_{SC}'s definition). We

New (Simpler!) Proof without AI!

establish the lemma via evaluating the integral $\mathbf{I}(\lambda,\theta)$ for both cases. But first note that for all settings in $\Theta^{e}_{\pm}$, the range of integration in $\mathbf{I}(\lambda,\theta)$ (viz. over the set M) is the same. This does not mean that we have presupposed that every $\mu\in M$ must be compatible with every $\theta\in\Theta^{e}_{\pm}$. For when we defined M, we allowed it to include apparatus existent quadruples which may be incompatible with some settings, or combinations thereof. And as earlier stressed, since these must have zero measure, they make no difference to the value of $\mathbf{I}(\lambda,\theta)$.

Consider now the upper sign case in (3.3), and a particular λ. (3.3) clearly implies that $\forall\theta\in\Theta^{e}_{+}$, $m_\mu(\lambda,\theta)[M^+] = 1$; where the measure $m_\mu(\lambda,\theta)[S]$ is defined (as usual) as $\int_S \rho(\mu/\lambda,\theta)d\mu$ $\forall$ subsets S of M, and $M^+ = \{\mu|\mathbf{ABCD}=+1\}$. Now, in general, M^+ will be a subset of M. But since $(M^+)'$ has measure-zero, we can define probabilities conditional on μ's outside M^+, but inside M, any way we like without affecting the value of $\mathbf{I}(\lambda,\theta)$. So we can choose to define these so that they also satisfy **ABCD**=+1. So far, then, we have $\forall\theta\in\Theta^{e}_{+},\forall\mu\in M$: **ABCD**=+1.

Now we proceed by 'separation of variables'. Fixing $\mu_{B,C,D}$ in M fixes the value of **A** in this set independently of its argument μ_A, since $\mathbf{A}(\lambda,\mu_A,\theta_A) = +1/[\mathbf{B}(\lambda,\mu_B,\theta_B)\mathbf{C}(\lambda,\mu_C,\theta_C)\mathbf{D}(\lambda,\mu_D,\theta_D)]$ throughout M. However, all μ_A in M must be compatible with each fixed triple of existents μ_B, μ_C, and μ_D in M, because the latter is a Cartesian product. (Recall from **2**, that we were free to choose M to be such a product without making any difference to the value of $\mathbf{I}(\lambda,\theta)$.) Thus $\mathbf{A}(\lambda,\mu_A,\theta_A)$ must be a constant throughout the set M - a constant we denote by $\mathbf{A}(\lambda,\theta_A)$. Continuing with the same reasoning yields that **B**, **C**, and **D** must be constants **B**, **C**, and **D**, $\forall\mu_B$, μ_C, and $\mu_D\in M$, respectively. Because of this constancy, we can take $\mathbf{I}(\lambda,\theta)$'s integrand outside of the integral, yielding:

$$\forall\lambda\in\Lambda_{TF}\cap\Lambda^{e}_{SC},\ \forall\theta\in\Theta^{e}_{+},\quad \mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D) = +1 \qquad (3.4)$$

because our initial choice of $\lambda\in\Lambda_{TF}\cap\Lambda^{e}_{SC}$ was arbitrary, and $m_\mu(\lambda,\theta)[M] = 1$ (by normalization). Using a similar argument, one can start from (3.3) with lower signs and show that:

$$\forall\lambda\in\Lambda_{TF}\cap\Lambda^{e}_{SC},\ \forall\theta\in\Theta^{e}_{-},\quad \mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D) = -1 \qquad (3.5)$$ QED.

Note that because of Lemma 2, not only have we obtained independence from apparatus existents, but throughout the set $\Lambda_{TF}\cap\Lambda^{e}_{SC}$ the stochastic framework we began with has collapsed into a deterministic one, for countably many strictly correlated and anti-correlated settings. For the fact that each of **A** through **D** lies in the interval [-1,+1], coupled with Lemma 2's constraint that their product equal ±1, implies that *each* of **A** through **D** must equal ±1 individually. It is also easily checked that Lemma 2's conclusion implies Shimony's TF, thus establishing that if TF and $|\Psi>$'s strict correlations hold in any λ-state, that state must also satisfy TF - as claimed at the end of section **2**.

Using the above lemmas, we can now prove the main theorem:

New Simpler Proof

New

Theorem - PL $\wedge$ SCRI $\Rightarrow$ $m_\lambda[(\Lambda_{TF})'] = 1$

Proof: - We need only show that $\Lambda_{TF} \cap \Lambda^c_{SC}$ is the empty set; for then $m_\lambda[\Lambda_{TF} \cap \Lambda^c_{SC}] = 0$, and by Lemma 1, the theorem follows. To show $\Lambda_{TF} \cap \Lambda^c_{SC}$ is empty, we show that there exists a choice of sets Θ^c_+ and Θ^c_- such that there cannot exist a $\lambda \in \Lambda_{TF} \cap \Lambda^c_{SC}$ satisfying both equations in Lemma 2. Choose:

$$\Theta^c_+ = \{<\theta_A,\theta_B,\theta_C,\theta_D>, <\theta_A-\pi/2,\theta_B+\pi/2,\theta_C,\theta_D>, <\theta_A,\theta_B+\pi/2,\theta_C+\pi/2,\theta_D>, <\theta_A+\pi/2,\theta_B,\theta_C+\pi/2,\theta_D>\}$$
$$\Theta^c_- = \{<\theta_A-\pi/2,\theta_B,\theta_C+\pi/2,\theta_D>\} \qquad (3.6)$$

where $<\theta_A,\theta_B,\theta_C,\theta_D>$ is an arbitrary quadruple in Θ_+. (The latter guarantees that Θ^c_+ and Θ^c_- are indeed subsets of Θ_+ and Θ_-, respectively, as required by their definition.) With these choices, Lemma 2 implies that each $\lambda \in \Lambda_{TF} \cap \Lambda^c_{SC}$ must satisfy:

$$\mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D) = +1 \qquad (3.7a)$$
$$\mathbf{A}(\lambda,\theta_A-\pi/2)\mathbf{B}(\lambda,\theta_B+\pi/2)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D) = +1 \qquad (3.7b)$$
$$\mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B+\pi/2)\mathbf{C}(\lambda,\theta_C+\pi/2)\mathbf{D}(\lambda,\theta_D) = +1 \qquad (3.7c)$$
$$\mathbf{A}(\lambda,\theta_A+\pi/2)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C+\pi/2)\mathbf{D}(\lambda,\theta_D) = +1 \qquad (3.7d)$$
$$\mathbf{A}(\lambda,\theta_A-\pi/2)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C+\pi/2)\mathbf{D}(\lambda,\theta_D) = -1 \qquad (3.7e)$$

By simple algebra, eqns. (3.7a-d) together imply:

$$\mathbf{A}(\lambda,\theta_A+\pi/2) = \mathbf{A}(\lambda,\theta_A-\pi/2) \qquad (3.8a)$$

while eqns. (3.7d,e) imply:

$$\mathbf{A}(\lambda,\theta_A+\pi/2) = -\mathbf{A}(\lambda,\theta_A-\pi/2) \qquad (3.8b)$$

Since all result functions must be deterministic (± 1), eqns. (3.8a,b) are contradictory. QED.

The above contradiction can be derived for many other choices of the sets Θ^c_+ and Θ^c_-. Some of these correspond to: (a) the uncountably many different ways to choose the setting quadruple $(\theta_A,\theta_B,\theta_C,\theta_D)$ defining the sets chosen in (3.6); (b) the arbitrariness in our choice to hold θ_D constant throughout $\Theta^c_+ \cup \Theta^c_-$ rather than one of the other three settings; (c) the arbitrariness in our choice to derive the contradiction in terms of the **A** result function rather than one of the other three result functions; and (d) the arbitrariness in the quadruple $<\theta_A,\theta_B,\theta_C,\theta_D>$ being taken to be an element of Θ_+, rather than Θ_- (choosing Θ_- interchanges our choices for the sets Θ^c_+ and Θ^c_- while preserving the contradiction: the +1's in eqns. (3.7) become -1's and vice versa).

These various ways to choose Θ^c_+ and Θ^c_- compatible with achieving a contradiction rule out the possibility that there may be contradictions only over a set of setting quadruples of measure-zero.

New

And so they help to make our theorem stronger than it stands at present. For we could have initially assumed that TF holds $\forall\theta\in\Theta^{c}_{+}\cup\Theta^{c}_{-}$ rather than $\forall\theta\in\Theta_{+}\cup\Theta_{-}$; and similarly weakened the universal quantifier in PL's definition (cf. **2**). With these modifications, the proof goes through analogously, and establishes that, with probability 1, at least one of Outcome Factorability or Outcome Locality must fail for at least one possible quadruple of apparatus existents, and measurement results; and for uncountably many setting quadruples satisfying $\theta_A + \theta_B - \theta_C - \theta_D = \pm\pi$ or 0 (not just for at least one such quadruple).

We shall now explain why GHZ's original proof is no longer valid in our measure-theoretic framework. As mentioned in **1**, GHZ start with the existence of deterministic result functions whose arguments encode the initial particles' existent, λ_0, back at the first decay. These functions are assumed independent of apparatus existents but satisfy OL; OF is trivially satisfied because it is implied by determinism. Invoking $|\Psi>$'s strict correlations, GHZ claim that:

$\exists\lambda_0\in\Lambda_0$ (=the set of all possible initial particle existents) such that:
$\mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D)=+1[-1]\ \forall\theta$ such that $\theta_A+\theta_B-\theta_C-\theta_D=\pi[0]$ (3.9)

without restricting this assertion to a countable number of setting quadruples which satisfy the right-hand equation, as in our Lemma 2. Indeed *with* such a restriction, their suggestion for a proof of contradiction cannot be carried through. For they say '...we can *continuously* vary two of the parameters [i.e. settings] while keeping the other two constant. This leads to the conclusion that **A=B=C=D**=constant. But this is impossible, since the product [i.e. on the left above] sometimes equals +1 and sometimes equals -1' (italics ours).

True, using a proof analogous to the final stage of ours (but slightly more involved) one can readily derive from (3.9) the conclusion that $\mathbf{A}(\lambda_0,\theta_A) = \mathbf{A}(\lambda_0,\theta_A+\delta)$ for arbitrary $\delta\in[0,2\pi)$ (and similarly for **B**-**D**) using continuously many strictly correlated setting quadruples (or continuously many strictly anti-correlated quadruples). And this implies that $\mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D)$ = constant[10] for all possible setting quadruples, in contradiction with (3.9)'s assertion that for some quadruples strict correlation must be predicted by λ_0, and for others strict anti-correlations must be predicted.

However, this proof turns on wrongly assuming that (3.9) follows from $|\Psi>$'s strict correlations without a 'countably many' restriction on settings satisfying the equations $\theta_A+\theta_B-\theta_C-$

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[10] We found no way to prove from (3.9) that all these result functions must be the *same* constant, as GHZ assert. In any case, a contradiction follows from the fact that each is *individually* a constant. Since writing this paper, we have checked that all the details of our interpretation of GHZ's suggested proof are correct by referring to Professor Greenberger's unpublished lecture notes (cf. eqns. (18)-(28) in Section VIII: 'Going Beyond Bell's Theorem—1. Efficient Counters'). New

$\theta_D = \pi[0]$. In the more specific class of theories envisaged by GHZ, these strict correlations must be recovered, given PL, by the integration:

$$\int_{\Lambda_0} \mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D)\rho(\lambda_0)d\lambda_0 = +1[-1], \forall\theta \text{ such that } \theta_A+\theta_B-\theta_C-\theta_D=\pi[0] \quad (3.10)$$

which is a special case of our SCRI. We shall simply give one of many counterexamples where (3.10) is true but (3.9), without restriction, is false.

Take Λ_0 to be a continuous set of existents; and suppose that (for either upper or lower signs):

$$\forall\theta\in\Theta_{\pm},\ \{\lambda_0|\mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D) = \pm 1\} = \Lambda_0 - \{\lambda_0(\theta)\} \quad (3.11)$$

where $\lambda_0(\theta)$ is a single element of Λ_0 - *which* element depending upon θ[11]. Then since $\forall\theta\in\Theta_{\pm}$, $m_{\lambda_0}[\Lambda_0 - \{\lambda_0(\theta)\}] = m_{\lambda_0}[\Lambda_0] = 1$:

$$\forall\theta\in\Theta_{\pm},\ m_{\lambda_0}[\{\lambda_0|\mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D)=\pm 1\}] = 1 \quad (3.12)$$

which makes (3.10) true. Suppose also that as θ varies throughout $\Theta_{\pm}$, $\lambda_0(\theta)$ varies throughout Λ_0. Then:

$$\begin{aligned}&\{\lambda_0|\mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D) = \pm 1,\ \forall\theta\in\Theta_{\pm}\} \equiv \\ &\quad \cap_{\theta\in\Theta_{\pm}}\{\lambda_0|\mathbf{A}(\lambda_0,\theta_A)\mathbf{B}(\lambda_0,\theta_B)\mathbf{C}(\lambda_0,\theta_C)\mathbf{D}(\lambda_0,\theta_D)=\pm 1\} = \\ &\quad\quad \cap_{\lambda_0(\theta)\in\Lambda_0}[\Lambda_0 - \{\lambda_0(\theta)\}] = \varnothing\end{aligned} \quad (3.13)$$

thus (3.9) is false.

In short, the final stage of our proof is more than just a simplification of GHZ's to a finite number of settings. This finiteness saves their proof from measure-theoretic invalidity. However, to be fair to GHZ, they do not use measures; since they launch their argument from EPR's premisses which equate measure-one with certainty (specifically, EPR's sufficient condition for identifying elements of reality). And it is easy to see that equating the two circumvents the trouble we have identified above. However, we regard it as interesting that without equating the two (which is, in any case, illegitimate), their proof needs to be modified in a nontrivial way to establish their result.

New!

Since we pointed out the above measure-theoretic invalidity in GHZ's proof, Greenberger *et*

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11 This does not mean that our counterexample somehow violates locality by making particle existents depend on settings. The notation '$\lambda_0(\theta)$' is merely meant to encode the fact that, in general, for different settings, different measure-one sets of λ_0 will predict strict correlations. This follows from the result functions depending on θ as well as λ_0, so that different settings may lead to different predictions dictated by the same λ_0-state.

New

et al.[5] have proposed an alternate proof of contradiction in order to establish their result. We would like to explain how the measure-theoretic trouble that occurs in GHZ's original proof is shared by this alternate proof. We are motivated to do this, partly because these authors explicitly claim that their proof goes through without equating measure-one with certainty, and partly by the fact that it allows us to explain the trouble in a different, perhaps more transparent, way.

The main difference between GHZ and Greenberger *et al.*[5] is that the latter (like us) aim to prove a contradiction directly in terms of one of the result functions, **A**, rather than all four. Using our notation, the main part of their proof starts from the assertion (cf. their eqn. (11a)):

$$\text{If } \theta_A+\theta_B-\theta_C-\theta_D = 0, \text{ then } \mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D) = -1 \qquad (3.14)$$

They then choose four appropriate quadruples of settings satisfying (3.14)'s antecedent, and deduce from it that:

$$\mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0), \text{ for a given angle } \phi \qquad (3.15)$$

But since the angle ϕ which figured in each of their setting quadruples can be chosen arbitrarily, they conclude (cf. their eqn. (16)):

$$\mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0) = \text{constant}, \forall\phi\in[0,2\pi) \qquad (3.16)$$

(3.16) then contradicts the equation:

$$\mathbf{A}(\lambda,\phi + \pi) = -\mathbf{A}(\lambda,\phi) \text{ for a given angle } \phi \qquad (3.17)$$

derived from another appropriately chosen setting quadruple satisfying the strictly correlated analogue of (3.14) (i.e. with π replacing 0, and +1 replacing -1).

We do not question the deduction of (3.16) *per se* - only Greenberger *et al.*'s claim (cf. Ref. 5, note 14) that it continues to go through if measure-one is not equated with certainty. For suppose we do not make this equation. Then all that one can infer from the strict anti-correlations $|\Psi>$ predicts is that the set of λ's for which (3.14)'s consequent is true has measure-one. And so, instead of (3.15), one can only derive:

$$\text{For a given angle } \phi\text{: } m_\lambda[\{\lambda|\mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0)\}] = 1 \qquad (3.18)$$

or, recognizing that the choice of ϕ was arbitrary:

$$\forall\phi\in[0,2\pi)\text{: } m_\lambda[\{\lambda|\mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0)\}] = 1 \qquad (3.19)$$

Our point is that one cannot derive (3.16), which (more explicitly) requires:

$$\exists\lambda \text{ such that } \forall\phi\in[0,2\pi): \mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0) = \text{constant} \qquad (3.20)$$

from (3.19). The reason is as before: For an uncountable infinity of ϕ's (such as all those in the range $[0,2\pi)$) it is perfectly possible to have:

$$\bigcap_{\text{uncountable set of } \phi} \{\lambda|\mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0)\} = \varnothing \; ! \qquad (3.21)$$

which would illustrate the falsity of (3.20), and *a fortiori* of (3.16).

However, by choosing a countable, or indeed finite, set of ϕ, this problem is easily avoided. Because if (3.19) is true:

$$m\lambda[\bigcap_{\text{countable set of } \phi} \{\lambda|\mathbf{A}(\lambda,2\phi) = \mathbf{A}(\lambda,0)\}] = 1 \qquad (3.22)$$

So there are plenty of values of λ for which $\mathbf{A}(\lambda,2\phi)$ = constant for any *countable* set of ϕ's. In fact we can use this to rescue Greenberger *et al* 's proof. For to get the contradiction they desire between (3.16) and (3.17), they do not need to use the full strength of the universal quantifier over uncountably many ϕ's in (3.16). In their derivation of (3.16) they could just specialize to $\phi = \pi/2$, while in deriving (3.17) they could specialize to $\phi = 0$, yielding the contradiction: $\mathbf{A}(\lambda,0) = -\mathbf{A}(\lambda,0)$. And since the choice of 0 is arbitrary, it is still possible to generate uncountably many such contradictions by running through the argument uncountably many times! But of course this then leads back to proving the contradiction in essentially the way that we ourselves have (cf. our contradiction between (3.8a) and (3.8b), and the discussion about strengthening our theorem that follows).

To sum up, we regard our own proof, discovered independently of GHZ's and Greenberger *et al*'s, as a nontrivial improvement on theirs. For it is surely logically possible that a measure-theoretically valid proof could not have been given - whereas our proof shows that the *physics* permits one to rule out this possibility.

4. COMPARISON WITH OTHER ALGEBRAIC PROOFS AND BELL'S PROOF

We shall first briefly compare section 3's proof to other algebraic proofs of nonlocality, before going on to compare it in more detail to Bell's statistical proofs. Our discussion will presuppose some acquaintence with the Bell-Kochen-Specker and Bell theorems (as discussed, for example, in Refs. 1, 11, 35 and 36).

The first published algebraic nonlocality proof was given by Heywood and Redhead (cf. Ref. 11, Ch. 6; and Refs. 37, 38). They realized that although the extension of Maczynski's theorem allows one to assign elements of reality to a composite quantum system as functions of locally maximal observables[39,40], one could nevertheless obtain a finite Bell-Kochen-Specker 'colouring'

contradiction for these systems by imposing, in addition to purely algebraic constraints on the elements of reality[12], a Bell-type locality condition. A simpler, more general proof was given by Stairs[42], and more recently Brown and Svetlichny[43]. They do not require *any* independent algebraic constraints on the elements of reality: they rely solely on a locality constraint and yet still arrive at a colouring contradiction.

Elby[44] has recently generalized their proof so that it applies to stochastic realist theories in which apparatus existents are causally relevant. Elby considers measurements undertaken on two correlated particles in the spin-1 singlet state, and exploits the strict correlations which obtain between the spin Hamiltonian of one particle and spin of the other to deduce a contradiction with TF and PL. As in our proof, locality assumptions together with the strict correlations force the realist theories at issue to 'EPR-collapse' to deterministic theories independent of apparatus existents (although the arguments we use to prove this collapse are somewhat different). Also, in both proofs the crucial final contradiction does not require any independent algebraic constraints. So both our proof and Elby's improve on previous proofs.

Further, Elby (like us) proves a contradiction between elements of reality (whose existence he *derives*) that pertain to just *one* of his two particles. However, Elby's proof still relies on the somewhat technical Bell-Kochen-Specker 'colouring' theorem which requires roughly 100 carefully chosen observables for which elements of reality exist, whereas our proof requires only 3 A-spin observables.

So much for how our proof improves upon other algebraic proofs. We now turn to discussing how it also improves on Bell's statistical proof. But first, it is important to be clear on how it doesn't improve on Bell's! We are motivated to do so in light of GHZ's claim that 'from the point of view of reality', their proof's restriction to the 'superclassical' (i.e. strict correlation) case makes it more interesting than Bell's. We disagree: our reason being that the same Bohrian response can be made to both proofs.

Both our proof and GHZ's (like Bell's) consider incompatible measurement contexts. In particular, the final contradictions are derived from statements about elements of reality (encoded by

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[12] Specifically, Heywood and Redhead assumed 'FUNC*', which in part requires that the elements of reality assigned to *maximal* observables mirror the functional relationships between the corresponding operators, but whose physical motivation was not fully clear. Elby[41] has recently derived FUNC* from other physically motivated assumptions. It is interesting to note that our eqn. (3.8b) is a consequence of FUNC*, and that our derivation of (3.8b) from locality and $|\Psi>$'s strict correlations corresponds in effect to a simplified version of Elby's derivation for that particular instance of FUNC*. This helps one see why our proof does not need to independently assume any algebraic constraints on elements of reality - for any constraints that *are* needed, are derived (complete with the elements of reality, themselves!) from locality and $|\Psi>$'s strict correlations.

the various result functions) that cannot all be displayed in a single measurement context. But we know from Bohr's original response to EPR[45] that he would reject any step in a proof which passes from one context to another incompatible context while retaining the definiteness of all elements of reality at issue. But just which step is this?

Consider first our proof. Since Bohr regarded QM as providing a complete description of physical reality, we can only legitimately talk of his response to a version of our proof that takes the QM state-vector to be (or, more weakly, to determine) an exhaustive specification of the particles' existent, λ. In that case (as mentioned in **2**) OF holds if and only if this state vector is a separable state. But $|\Psi>$ is manifestly nonseparable, Bohr no doubt would have denied OF. (And since Bohr writes that 'any direct mechanical interaction of the system and the measuring agencies is excluded'[46], thus ruling out manifestly causal nonlocal connections, he presumably would endorse OL[33].) More specifically, our contradiction arises from assuming that OF holds for all contexts regardless of whether or not they are compatible. So in the case where $\lambda = |\Psi>$ (the only case Bohr would recognize as legitimate), we effectively assume the particles' joint state divides up into parts once and for all, in a context-independent way. This is opposed to Bohr's response to EPR in which he effectively stresses that the particles' state 'splits' up into parts in a way that depends on the measurement context. (Put in terms of wavepacket reduction - the basis in which reduction occurs for the joint 4 particle system depends on which measurements are performed.)

As for GHZ's own proof, they preface it with EPR's original (less formal) argument and so inherit Bohr's objection to the ambiguity in their phrase 'disturbance' - which could mean either the 'splitting' induced by the measurement context discussed above, which Bohr accepts, or a more obviously 'causal' violation of OL, which he rejects. Admittedly, it is tempting to think that Bohr's position is more tenuous because all the measurement contexts GHZ consider admit strict correlations, and so elements of reality. However, Bohr would hold that once a certain context is realized and elements of reality attributed in that context, this blocks attributing elements of reality to other incompatible contexts which aren't in fact realized[13].

Now to improvements on Bell's theorem. Famously, it requires only two of the four particles in our *Gedankenexperiment* while our proof required at least three[14]. But there is a link.

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[13] Since writing this paper, Greenberger *et al.* (Ref. 5, note 10) have themselves stressed that Bohr would reject their argument, in complete agreement with what we've said here.

[14] Indeed, since both our theorem and Bell's can be applied without using all particles, our proofs remain valid using weakened locality assumptions. For example, OF can be weakened so that only one (in Bell's case) or two (in ours) of the four outcomes is 'factored' out of OF's conditional joint probability for the four outcomes. Similar weakenings are possible for OL and PL, but we shall not pursue this matter here in order to keep things simple.

Suppose this *Gedankenexperiment* is performed with $\theta_{B,D}$ constant throughout so that the correlation coefficient predicted by $|\Psi>$ reduces to $-\cos(\theta_A - \theta_C + \Delta)$, where Δ $(= \theta_B - \theta_D)$ is a constant: call this correlation $CC_{|\Psi>}(\theta_A,\theta_C,\Delta)$. It is well-known that there exist *many* quadruples $\theta_A,\theta_A',\theta_C,\theta_C'$ (no pair of which need admit strict correlations) such that[47]:

$$|CC_{|\Psi>}(\theta_A,\theta_C,\phi)+CC_{|\Psi>}(\theta_A,\theta_C',\phi)+CC_{|\Psi>}(\theta_A',\theta_C,\phi)-CC_{|\Psi>}(\theta_A',\theta_C',\phi)| = 2 + f(\theta_A,\theta_A',\theta_C,\theta_C',\phi) > 2 \quad (4.1)$$

thus allowing Bell's theorem to be applied to the A and C particles.

In its most general form (Ref. 1, Ch. 7) this theorem begins by adopting OF, OL, and PL, $\forall\lambda\in\Lambda,\forall\theta_A,\theta_A',\theta_C,\theta_C'\in\Gamma$ (where $\Gamma = \{\theta_A,\theta_A',\theta_C,\theta_C' | f > 0\}$). In addition, Bell makes two further assumptions, motivated by locality, concerning the apparatus existent distributions:

$\forall\mu,\lambda,\theta_A,\theta_A',\theta_C,\theta_C'\in\Gamma$--

Apparatus Factorability (AF): $\rho(\mu/\lambda,\theta)=\rho(\mu_A/\lambda,\theta)\rho(\mu_B/\lambda,\theta)\rho(\mu_C/\lambda,\theta)\rho(\mu_D/\lambda,\theta)$

Apparatus Locality (AL): $\rho(\mu_A/\lambda,\theta)=\rho(\mu_A/\lambda,\theta_A);\rho(\mu_B/\lambda,\theta)=\rho(\mu_B/\lambda,\theta_B);$
$\rho(\mu_C/\lambda,\theta)=\rho(\mu_C/\lambda,\theta_C);\rho(\mu_D/\lambda,\theta)=\rho(\mu_D/\lambda,\theta_D)$

We shall call the conjunction of these assumptions 'Total Apparatus Factorability' (TAF).

In precise analogy to OF and OL, the probability densities $\rho(\mu_A/\lambda,\theta)$, etc. appearing in AF and AL are defined in the usual way as marginals obtained from the distribution $\rho(\mu/\lambda,\theta)$; and the distributions on the right-hand side of AL's equations are defined as the common value of the left-hand sides as the settings not appearing on the right vary. Further, as discussed in **2**, TAF is most plausible when its distributions are taken to be already averaged over all other existents in the spacetime regions to the past of the four measurement interactions.

With these assumptions, the integrand in SCRI's integral (which gives the existent expression for the four spin correlation prior to λ-averaging) reduces to $\underline{\mathbf{A}}(\lambda,\theta_A)\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C)\underline{\mathbf{D}}(\lambda,\theta_D)$, where $\underline{\mathbf{A}}(\lambda,\theta_A) = \int_{M_A}\mathbf{A}(\lambda,\mu_A,\theta_A)\rho(\mu_A/\lambda,\theta_A)d\mu_A$, etc. (Note that since, without loss of generality, M can be regarded as the Cartesian product $M_A\times M_B\times M_C\times M_D$, Bell can indeed factor SCRI's integral into four integrals of this type.) Using the fact that $|\mathbf{A}|,|\mathbf{B}|,|\mathbf{C}|$, and $|\mathbf{D}| \le 1$ for all values of their arguments, Bell's inequality can thus be derived: ← New

$$\forall\theta_A,\theta_A',\theta_C,\theta_C'\in\Gamma,\ |\underline{\mathbf{A}}(\lambda,\theta_A)\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C)\underline{\mathbf{D}}(\lambda,\theta_D)+\underline{\mathbf{A}}(\lambda,\theta_A)\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C')\underline{\mathbf{D}}(\lambda,\theta_D) +\underline{\mathbf{A}}(\lambda,\theta_A')\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C)\underline{\mathbf{D}}(\lambda,\theta_D) - \underline{\mathbf{A}}(\lambda,\theta_A')\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C')\underline{\mathbf{D}}(\lambda,\theta_D)| \le 2 \quad (4.2)$$

Multiplying (4.2) by $\rho(\lambda)$ and integrated over Λ constrains the (actually measured) λ-averaged correlations in contradiction with (4.1).

One difference between Bell's theorem and ours is that his yields a statistical contradiction, whereas ours leads to an algebraic one. So in one respect, our conclusion is stronger. For the strongest conclusion that can be deduced from the contradiction between (4.1) and (4.2) is that at least one of Bell's locality assumptions must be violated for a measure of λ-states of *at least* f/2. Since the maximum value f can take is $2(\sqrt{2} - 1)$[48], this measure's highest value is just over 2/5: much less than the measure 1 set of λ-states required by our theorem to violate our locality assumptions[15]. So our theorem goes beyond Bell's by proving that realist theories able to reproduce all QM's predictions must violate locality assumptions (given appropriately chosen settings) in *all* but a measure zero of trials; as in the de Broglie-Bohm theory[21] where the quantum potential is never 'turned off' from one trial to the next. (By contrast, one can easily show that the simple model devised by Bell (Ref. 1, pp. 16 - 17) violates OL for no more than the minimum measure of λ-states, f/2, required by Bell's theorem. Thus our theorem shows it is incapable of reproducing the full range of $|\Psi>$'s 4 particle correlations without significant modification.)

A second difference is that our proof does not require the extra assumptions AF and AL, since we derive the independence of outcomes from apparatus existents via our other assumptions without imposing any conditions on their distributions (cf. **3**, Lemma 2). Physically, this means that our theorem rules out a class of apparatus nonfactorable [and apparatus nonlocal] theories which, while satisfying OL so that each A-result depends only upon the μ_A-existent (and λ and θ_A) in the immediate causal history of this result, violate AF [or AL] so that the probability of μ_A's occurrence depends upon (say) the distant C-apparatus existent [or setting].

True, the motivation for assuming AF and AL is the same as that for assuming OL. But as always, it is worth weakening the locality assumptions used in these proofs, so as to get stronger conclusions about what kind of nonlocality they require. And although there are no clearcut examples in the literature of TAF-violating theories where OL holds, such theories are far from vacuous. In particular, a μ_A-$\mu_C[\text{-}\theta_C]$ stochastic dependence of the kind prohibited by AF [AL] is compatible with the assumption (contained in OL) that the A-result is stochastically independent of μ_C[and θ_C] *for fixed* μ_A. For one must distinguish between: μ_C[or θ_C] and μ_A *independently* causally influencing the A-result statistics (i.e. two distinct causes of a common effect) which OL prohibits; and μ_C[or θ_C] causally influencing these statistics *via* μ_A (i.e. a two-step causal chain) which OL allows[16].

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15 It may be possible to improve on Bell's measure by considering higher-order Bell-type 'facet' inequalities[49], or 'chained' Bell inequalities[50].

16 Ruling out the first type of dependence while allowing the second is analogous to imposing the restriction on the function F(x(y),y) that it equal F(x(y)) $\forall$y, while continuing to allow x itself to depend upon y. Indeed, classical deterministic analogues of these two types of (causal) dependence abound. Recall that: (I) the force on a surface from which two (massless) Hookean springs with spring constants k_X and k_Y are suspended in parallel is $F(x,y) = k_X x +$

It is of course true that Bell could have avoided TAF by reinstating in his proof enough strict correlations to permit a derivation much like ours of the independence of outcomes from apparatus existents. (Indeed, Bell's informal EPR argument for determinism in his early proof (Ref. 1, Ch. 2) comes close to doing this, although he makes no explicit reference to apparatus existents, nor to their distributions.) On the other hand, Bell would then be sacrificing one of the main merits of his later proof - viz. its not requiring experimentally unrealizable strict correlations. We show in the next section that our proof shares this merit once TAF is added to its assumptions.

5. THE NONSTRICTLY CORRELATED VERSION OF THE PROOF AND A NEW TESTABLE 'QUASI-ALGEBRAIC' LOCALITY INEQUALITY

The following is a nonstrictly correlated version of section 3's algebraic nonlocality proof which applies to roughly the same class of theories as does Bell's nonstrictly correlated proof, and also leads to a testable locality inequality.

For the theories at issue we must weaken section 2's SCRI to:

Non Strict Correlation Recovery Integral (NSCRI):

$$\int_\Lambda \left[\int_M \left(\sum_\varepsilon \Pi_\varepsilon P(\varepsilon/\lambda,\mu,\theta)\right)\rho(\mu/\lambda,\theta)d\mu\right]\rho(\lambda/\theta)d\lambda = CC(\theta) \quad \forall\theta$$

where we can either:

(I) take CC(θ) to be the experimentally observed correlation for a θ for which QM predicts strict correlations (the difference due to inefficiencies of spin-meters - cf. Ref. 1, pp. 86-7; and 46, p. 1889); or
(II) take CC(θ) to be a QM-predicted correlation for a θ for which QM does not predict strict correlations (either in the case considered up to now, where the particles travel in precisely opposite directions along the z-axis; or in the more practical case where they don't, in which case QM does not predict strict correlations in state $|\Psi>$ anyway).

In either case $CC(\theta) \notin \{0,1\}$. Now we define:

$$\Lambda_{NSC} \equiv \{\lambda | I(\lambda,\theta) > 0,\ \forall\theta\in\Theta^4 \text{ and } I(\lambda,\theta_5) < 0\}$$

where, as in 3, $I(\lambda,\theta)$ is a shorthand for NSCRI's apparatus existent integral; and:

$$\Theta^4 = \{<\theta_A,\theta_B,\theta_C,\theta_D>, <\theta_A',\theta_B',\theta_C,\theta_D>, <\theta_A,\theta_B',\theta_C',\theta_D>, <\theta_A'\pm\pi,\theta_B,\theta_C',\theta_D>\} \equiv \{\theta_1,\theta_2,\theta_3,\theta_4\}$$

$$\theta_5 \equiv <\theta_A',\theta_B,\theta_C',\theta_D>$$

$k_y y$, where x and y are their extensions; while (II) the force with the same two springs suspended in series is $F(x(y)) = k_x x$, where $x = (k_y/k_x)y$.

where $\theta_A,\theta_A',\theta_B,\theta_B',\theta_C,\theta_C'$, and θ_D are arbitrary settings. (As in **3**, there are many ways to structure one's choice of settings compatible with a valid proof of contradiction; but, for simplicity, we shall not attempt to account for them all.)

We present the proof in two stages, beginning with a lemma:

Lemma - PL $\wedge$ NSCRI $\Rightarrow$ $m_\lambda[\Lambda_{NSC}] \geq CC(\theta_1) + CC(\theta_2) + CC(\theta_3) + CC(\theta_4) - CC(\theta_5) - 4$

Proof: - Define $\Lambda_{>0}(\theta) \equiv \{\lambda | \mathbf{I}(\lambda,\theta) > 0\}$ and, similarly, $\Lambda_{<0}(\theta) \equiv \{\lambda | \mathbf{I}(\lambda,\theta) < 0\}$. By NSCRI and PL:

$$\forall\theta,\ CC(\theta) = \int_\Lambda \mathbf{I}(\lambda,\theta)\rho(\lambda)d\lambda = \int_{\Lambda_{>0}(\theta)} \mathbf{I}(\lambda,\theta)\rho(\lambda)d\lambda + \int_{\Lambda_{<0}(\theta)} \mathbf{I}(\lambda,\theta)\rho(\lambda)d\lambda$$

$$> \quad 0\cdot m_\lambda[\Lambda_{>0}(\theta)] \quad + \quad -1\cdot m_\lambda[\Lambda_{<0}(\theta)] \qquad (5.1)$$

$$\text{and} \quad < \quad 1\cdot m_\lambda[\Lambda_{>0}(\theta)] \quad + \quad 0\cdot m_\lambda[\Lambda_{<0}(\theta)]$$

Thus $m_\lambda[\Lambda_{>0}(\theta)] > CC(\theta)$ and $m_\lambda[\Lambda_{<0}(\theta)] > -CC(\theta)$, $\forall\theta$. In particular, $m_\lambda[\Lambda_{>0}(\theta)] > CC(\theta)$ $\forall\theta\in\Theta^4$, and $m_\lambda[\Lambda_{<0}(\theta_5)] > -CC(\theta_5)$. Using this, the lemma follows by applying (for n=5) the following theorem of probability theory (easily proved by induction):

If $m_\lambda[S_1] \geq q_1$, $m_\lambda[S_2] \geq q_2$, ..., $m_\lambda[S_n] \geq q_n$, ... where $S_1, S_2, \ldots S_n, \ldots$ is a countable sequence of sets, then $m_\lambda[\cap_n S_n] \geq (\Sigma_n q_n) - (n-1)$ QED.

The idea of the theorem is to put a lower limit on the measure of particle existents which must violate either Total Factorability, or Total Apparatus Factorability, for at least one of the settings $\theta_1,\theta_2,\theta_3,\theta_4$, or θ_5, and at least one quadruple of apparatus existents and measurement results. So we define:

$$\Lambda_{TF\wedge TAF} = \{\lambda | \forall\mu\in M, \forall\theta\in\Theta^4\cup\{\theta_5\}, \forall\varepsilon\text{: TF and TAF hold}\}$$

in order to put a lower limit on this set's complement. (Clearly the theorem is only significant as a proof of nonlocality if this lower limit is positive - more on this below.)

Theorem - PL $\wedge$ NSCRI $\Rightarrow$ $m_\lambda[(\Lambda_{TF\wedge TAF})'] \geq CC(\theta_1) + CC(\theta_2) + CC(\theta_3) + CC(\theta_4) - CC(\theta_5) - 4$

Proof: - It suffices to show that $\Lambda_{TF\wedge TAF}\cap\Lambda_{NSC}$ must be the empty set. For then we have three results which together imply the theorem: (1) $m_\lambda[(\Lambda_{TF\wedge TAF})'] \geq m_\lambda[\Lambda_{NSC}] - m_\lambda[\Lambda_{TF\wedge TAF}\cap\Lambda_{NSC}]$ (since for any sets S_1 and S_2, $m_\lambda[S_1'] = 1 - m_\lambda[S_1] \geq m_\lambda[S_1\cup S_2] - m_\lambda[S_1] = m_\lambda[S_2] - m_\lambda[S_1\cap S_2]$); (2) $m_\lambda[\Lambda_{TF\wedge TAF}\cap\Lambda_{NSC}] = 0$; and (3) the Lemma result.

By the very definition of the set $\Lambda_{TF\wedge TAF}\cap\Lambda_{NSC}$, each λ in it satisfies $\mathbf{I}(\lambda,\theta) = \mathbf{A}(\lambda,\theta_A)\mathbf{B}(\lambda,\theta_B)\mathbf{C}(\lambda,\theta_C)\mathbf{D}(\lambda,\theta_D)\forall\theta\in\Theta^4\cup\{\theta_5\}$, where $\mathbf{A}(\lambda,\theta_A) = \int_{M_A} \mathbf{A}(\lambda,\mu_A,\theta_A)\rho(\mu_A/\lambda,\theta_A)d\mu_A$, and similarly for the other three μ-averaged result functions. Furthermore, each $\lambda \in \Lambda_{TF\wedge TAF}\cap\Lambda_{NSC}$

must satisfy:

$$\underline{\mathbf{A}}(\lambda,\theta_A)\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C)\underline{\mathbf{D}}(\lambda,\theta_D) = k_1 > 0 \quad (5.2a)$$

$$\underline{\mathbf{A}}(\lambda,\theta_A')\underline{\mathbf{B}}(\lambda,\theta_B')\underline{\mathbf{C}}(\lambda,\theta_C)\underline{\mathbf{D}}(\lambda,\theta_D) = k_2 > 0 \quad (5.2b)$$

$$\underline{\mathbf{A}}(\lambda,\theta_A)\underline{\mathbf{B}}(\lambda,\theta_B')\underline{\mathbf{C}}(\lambda,\theta_C')\underline{\mathbf{D}}(\lambda,\theta_D) = k_3 > 0 \quad (5.2c)$$

$$\underline{\mathbf{A}}(\lambda,\theta_A'\pm\pi)\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C')\underline{\mathbf{D}}(\lambda,\theta_D) = k_4 > 0 \quad (5.2d)$$

$$\underline{\mathbf{A}}(\lambda,\theta_A')\underline{\mathbf{B}}(\lambda,\theta_B)\underline{\mathbf{C}}(\lambda,\theta_C')\underline{\mathbf{D}}(\lambda,\theta_D) = k_5 < 0 \quad (5.2e)$$

By simple algebra, eqns. (5.2a-e) together imply:

$$[\underline{\mathbf{A}}(\lambda,\theta_A)]^2 = k_1k_3[\underline{\mathbf{A}}(\lambda,\theta_A')]^2/k_2k_5 \quad (5.3)$$

Since, by eqns. (5.2a-e), each μ-averaged result function must be nonzero and k_1k_3/k_2k_5 must be negative, (5.3) is a contradiction - the left-hand side must be positive while the right-hand side negative. QED.

The final stage of this proof is analogous to the final stage of section **3**'s strictly correlated proof. It differs mainly in our use of Bell's strategy (cf. **4**) of first averaging over apparatus existents, using TAF. Also, in contrast to section **3**, there is no collapse to determinism, and the contradiction arises not between Einsteinian elements of reality, but between probabilities for measurement results. For this reason, the proof is best referred to as a 'quasi-algebraic' proof. But note that the proof reduces to a slightly weaker version of our strictly correlated proof if one takes QM to predict strict correlations for the four quadruples in Θ^4, and strict anti-correlation for θ_5. In that case, $CC(\theta_1) = CC(\theta_2) = CC(\theta_3) = CC(\theta_4) = +1$ and $CC(\theta_5) = -1$, so that the conclusion becomes $m_\lambda[(\Lambda_{TF\wedge TAF})'] = 1$: differing only from our strictly correlated proof in the inclusion of TAF as one of the locality principles whose conjunction must be false with probability 1.

We now discuss the significance of this nonstrictly correlated proof. Consider first case (I), where $CC(\theta)$ is the experimentally observed correlation when θ should, according to QM, display strict correlation or anti-correlation. Clearly the above theorem necessitates a renunciation of locality for a nonzero-measure of particle existents only if the following inequality is observed:

Quasi-Algebraic Locality Inequality. $\quad CC(\theta_1) + CC(\theta_2) + CC(\theta_3) + CC(\theta_4) + (-CC(\theta_5)) > 4$

A necessary condition for this inequality to hold is that at least one of the terms summed on the left-hand side must empirically be shown to be > 0.8. So if, as in the previous paragraph, θ_1 through θ_4 permit QM strict correlations and θ_5 anti-correlation - so that the first four terms above would, under ideal experimental conditions, be expected to be +1, and the last -1 - our theorem allows for at least a 20% deviation from these theoretical predictions due to nonideal conditions[17].

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[17] In general, our proof allows for at least a [100/N]% deviation from theoretical strict correlations, where N is the number of setting quadruples used to derive the contradiction. It is thus advantageous that N can be made low: e.g. in the text, N=5. For with other algebraic proofs relying on the Bell-Kochen-Specker contradiction, $N \approx 100$ [36,51-54]. We conjecture that this will seriously impede their extension to the nonstrictly correlated case from the experimental

Consider now case (II) where each $E(\theta)$ is a QM-predicted nonstrict correlation. Clearly many choices of θ_1 through θ_5 satisfy the inequality, but for simplicity let us choose:

$$\theta_A+\theta_B-\theta_C-\theta_D = \theta_A'+\theta_B'-\theta_C-\theta_D = \theta_A+\theta_B'-\theta_C'-\theta_D = (\theta_A'\pm\pi)+\theta_B-\theta_C'-\theta_D \qquad (5.4)$$

so that the theorem then says $m_\lambda[(\Lambda_{TF\wedge TAF})'] \geq -5\cos(\theta_A+\theta_B-\theta_C-\theta_D) - 4$. An interesting question arises: Does our theorem necessitate a measure of λ-states which must violate either TF or TAF that is lower than Bell's?

As mentioned in **4**, the strongest conclusion the Bell theorem can reach is $m_\lambda[(\Lambda_{TF\wedge TAF})'] \geq \sqrt{2} - 1$; whereas this lower limit is improved upon by our theorem (with the general choice of settings above) for a whole range of settings - specifically when $208° > \theta_A+\theta_B-\theta_C-\theta_D > 151°$. On the other hand, if both strength of conclusion (i.e. *high* measure of states violating locality) and the difficulty of measuring tight correlations are considered together, Bell's theorem fares better. For the magnitude of the measured correlation Bell requires to establish the lower bound $\sqrt{2} - 1$ is $|-\cos(\pi/4)| \approx 0.71$; whereas for our proof to *improve* on this lower bound one would need to measure a correlationmagnitudeof $|-\cos(\theta_A+\theta_B-\theta_C-\theta_D)| > (\sqrt{2} + 3)/5 \approx 0.88$, 24% greater than 0.71. In any case, if the main experimental interest is simply to establish that the measure of locality-violating states must be nonzero, our theorem compares more favourably to Bell's in only requiring the capability of measuring a 0.80 correlation.

It remains to be seen how these theorems compare with respect to other practical difficulties: e.g. the need for assumptions like Clauser and Horne's[58] 'no-enhancement', and whether such assumptions are more plausible than their Bell theorem counterparts. Such issues would interest those who continue to be sceptical about the empirical warrant for nonlocality based on recent Bell experiments[59]. For our part, although our nonstrictly correlated proof is of practical interest, at a philosophical level we concur with GHZ who say: 'Our general attitude is that we assume that quantum mechanics gives correct answers, and the question is whether a classical model can reproduce these answers' (p. 71). We agree for two reasons. First, prior to experimental tests of the singlet state's correlations, there was a clear need to develop a nonstrictly correlated version of Bell's theorem so as to adapt it to experiment. But since these experiments have overwhelmingly confirmed the singlet state's nonstrict correlations, they already provide strong inductive support for its *strict*

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point of view. Furthermore, these other proofs face the extra practical difficulty of measuring the spin Hamiltonian[55]. (Since writing this paper, our conjecture has been confirmed by Elby[56] who gives an elegant nonstrict correlations version of his Bell-Kochen-Specker-type algebraic proof (cf. 4). However, Elby argues that his proof has nontrivial implications, not for experiments, but for a conceptual dilemma it poses for the realist: either give up locality, or deny that conservation laws can hold *even approximately* (as in the case of CP violation). For an analysis and strengthening of Elby's argument which supports his conclusion, cf. Clifton[57].)

New!

correlations, and furthermore for the *strict* correlations predicted by QM in general: in particular for |Ψ>'s. Second, *a priori* there is already good reason to accept the existence of strict correlations. For they are grounded in fundamental conservation principles which we have reason to believe quite apart from our reasons for believing in QM.

Finally, it is worth noting that our quasi-algebraic inequality has in effect been independently discovered by Greenberger *et al.*[5]. ('In effect', because they derive, in a somewhat different way, a 3 particle version of our inequality by considering a simpler QM state of the type recently discovered by Mermin - cf. note 5.) As in our proof, Greenberger *et al.* drop QM's strict correlations, and so drop EPR's argument which can only be launched from these correlations. However, their derivation *does* assume what they call 'EPR's program', which for them amounts to assuming the existence of deterministic result functions with no causal dependence on apparatus existents. In contrast, our derivation does not tie itself down to 'EPR's program', in this sense, and so our inequality is capable of experimentally falsifying a weaker EPR program than the one they postulate. And we believe it really is weaker, despite the fact of Stapp[60] and Fine[61] having general proofs of the equivalence of stochastic and deterministic models (cited by Greenberger *et al.*).

In brief, our reason is this. Recall:- Fine's reduction of a factorable stochastic model - i.e. a model which gives probabilities for results A, B, etc. conditional on λ and quantities a, b, etc. (i.e. pr(A/λ,a), pr(A,B/λ,a,b), etc.) - to a deterministic model goes by defining the big joint p(A,A',...,B,B',.../a,a',...,b,b',...) in terms of the factorable stochastic model, as:

$$\Sigma_\lambda \, pr(A/\lambda,a)pr(A'/\lambda,a')...pr(B/\lambda,b)pr(B'/\lambda,b')...pr(\lambda) \qquad (5.5)$$

He then easily shows in the usual way that this big joint is equivalent to a deterministic model: each λ is a sequence of results <A,A',...,B,B',...>, etc. Now, given the big joint, the deterministic model is uniquely fixed. But given the stochastic model, many big joints exist whose corresponding deterministic model delivers the same statistics as the stochastic model. (Fine's choice above has the particular property that at each λ, any values A, B of any 2 quantities a, b are stochastically independent.) So one is at liberty to deny the joints' physical reality - at least until someone gives an account of physically real probability which singles out just 1 big joint. (Cf. Butterfield[62] for further discussion of this point.)

New (Essentially from our last letter to Shimony)

6. CONCLUSION

Our main contribution in this paper has been to provide a quite general measure-theoretically valid proof of nonlocality along the lines of Greenberger, Horne, and Zeilinger. Along the way we discussed a number of related issues, including the plausibility of our assumptions (especially in light of the EPR-Bohr debate), and the proof's relationship to other algebraic nonlocality proofs. We also discussed the advantages of our proof over Bell's, and derived a new testable quasi-algebraic locality

inequality. Although this last result is of practical interest, nevertheless in our view it is subordinate to section 3's proof, which invokes correlations that already have inductive confirmation, and are motivated by conservation principles. We believe this proof makes the case against local realism the strongest it has been since John Bell's ground-breaking work over 25 years ago.

APPENDIX - INDEX OF ABBREVIATIONS USED

AF Apparatus Factorability: $\rho(\mu/\lambda,\theta)=\rho(\mu_A/\lambda,\theta)\rho(\mu_B/\lambda,\theta)\rho(\mu_C/\lambda,\theta)\rho(\mu_D/\lambda,\theta)$

AL Apparatus Locality: $\rho(\mu_A/\lambda,\theta)=\rho(\mu_A/\lambda,\theta_A)$; $\rho(\mu_B/\lambda,\theta)=\rho(\mu_B/\lambda,\theta_B)$; $\rho(\mu_C/\lambda,\theta)=\rho(\mu_C/\lambda,\theta_C)$; $\rho(\mu_D/\lambda,\theta)=\rho(\mu_D/\lambda,\theta_D)$

EPR Einstein, Podolsky, and Rosen

GHZ Greenberger, Horne, and Zeilinger

NSCRI NonStrict CorrelationRecovery Integral:

$$\int_\Lambda\left[\int_M\left(\sum_\varepsilon\Pi_\varepsilon P(\varepsilon/\lambda,\mu,\theta)\right)\rho(\mu/\lambda,\theta)d\mu\right]\rho(\lambda/\theta)d\lambda = CC(\theta)\ \ \forall\theta \quad \text{with } CC(\theta)\notin\{0,1\}$$

OF Outcome Factorability: $P(\varepsilon/\lambda,\mu,\theta) = P(\varepsilon_A/\lambda,\mu,\theta)\,P(\varepsilon_B/\lambda,\mu,\theta)\,P(\varepsilon_C/\lambda,\mu,\theta)\,P(\varepsilon_D/\lambda,\mu,\theta)$

OF_0 Outcome Factorability, with probabilities conditional on initial existents (λ_0,μ_0)

OL Outcome Locality: $P(\varepsilon_A/\lambda,\mu,\theta) = P(\varepsilon_A/\lambda,\mu_A,\theta_A)$; $P(\varepsilon_B/\lambda,\mu,\theta) = P(\varepsilon_B/\lambda,\mu_B,\theta_B)$ $P(\varepsilon_C/\lambda,\mu,\theta) = P(\varepsilon_C/\lambda,\mu_C,\theta_C)$; $P(\varepsilon_D/\lambda,\mu,\theta) = P(\varepsilon_D/\lambda,\mu_D,\theta_D)$

PL Particle Locality: $\rho(\lambda/\theta) = \rho(\lambda)\quad\forall\lambda,\theta\in\Theta$

QM Quantum Mechanics

TAF Total Apparatus Factorability: Conjunction of Apparatus Factorability, and Apparatus Locality

TF Total Factorability: Conjunction of Outcome Factorability, and Outcome Locality

SCRI Strict CorrelationRecovery Integral:

$$\int_\Lambda\left[\int_M\left(\sum_\varepsilon\Pi_\varepsilon P(\varepsilon/\lambda,\mu,\theta)\right)\rho(\mu/\lambda,\theta)d\mu\right]\rho(\lambda/\theta)d\lambda = \pm1\ \ \forall\theta\in\Theta_\pm$$

New!

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